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PROSPECTS FOR GRAVITATIONAL PHYSICS EXPERIMENTS WITH CRYOGENIC TORSION BALANCES

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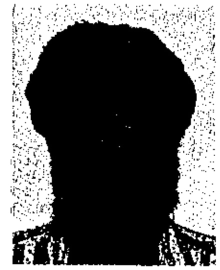

Abstract

We discuss prospects for improved tests of the weak equivalence principle, searches for anomalies in Newtonian gravity (or new forces), and measurements of the gravitational constant G, using torsion balances operating at cryogenic temperatures. Operation at low temperature promises the benefits of high mechanical Q with correspondingly low thermal noise, high mechanical stability, and high temperature stability. We discuss potential problems in a G measurement due to fiber non-linearity and/or inelasticity, and the problem of minimizing Newtonian gravitational couplings to a balance used for an equivalence principle test. Sensitivity to a differential acceleration of test masses at a level 10^{-15} cm/s^2 or less appears to be a reasonable goal.

1. Introduction

The classic torsion balance is remarkable in its ability to measure extremely weak slowly varying forces, but is nearing its limits when operated at room temperature. The natural next step is to develop torsion balances which operate at cryogenic temperatures. In a previous publication[1] we have discussed limits on such instruments which might be imposed by thermal, seismic, and readout noise sources, concluding that these limitations are consistent with the measurement of differential accelerations of test masses with sensitivity $\Delta a = 10^{-15}$ cm/s^2 or better using an instrument operating at a temperature of about 4K. Such measurements could improve current limits on the universality of free fall in gravitational fields of the sun, earth, and/or local mass sources by three orders of magnitude or more. In the present paper we address other potential limitations on the performance of torsion balances, especially Newtonian gravitational couplings and possible effects of nonlinear and anelastic behavior of torsion fibers.

2. Choice of fiber material

To minimize thermal noise a fiber material with the highest possible torsional oscillation Q is desired. Materials we have considered include: *tungsten*, which is strong and widely used for room temperature torsion balances; *sapphire*, which in bar form exhibits extremely high Q at low temperatures and is available in fiber form with diameter as small as 75 μm (larger than ideal); *aluminum 5056*, which in bar form also exhibits high Q at low temperatures, and is favored for use in gravitational wave bar detectors; and *BeCu*, favored by condensed matter experimentalists for torsion pendulums operated at high frequencies and low temperature. Figure 1 displays Q's measured in our lab, as a function of oscillation amplitude, for pendulums operating at about 4K using 25 cm long fibers of: tungsten (diameter d = 20 μm, period τ_0 = 268 s), hardened aluminum 5056 (d = 50 μm, τ_0 = 105 s), and sapphire (d = 70 μm, τ_0 = 97 s). We are currently testing a BeCu fiber. Fused silica, the fiber material of choice for room temperature instruments, is

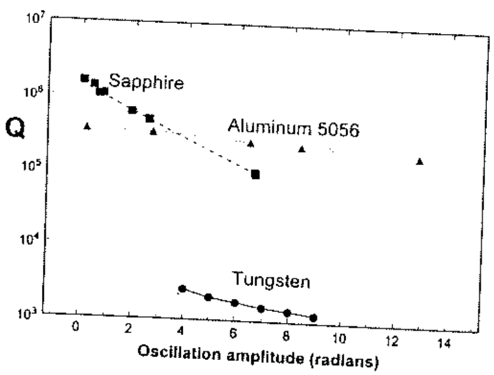

Fig. 1: Measured Q's of torsion pendulums using various fiber materials, as a function of oscillation amplitude at temperatures near 4.2K.

interestingly very unsuitable for use at low temperature, where the temperature coefficient of its shear modulus (and hence its torsion constant) is an order of magnitude greater than at room temperature, while its Q is expected to be little better than at room temperature.

It is surprising that our measured fiber Q's are not higher. For small amplitude oscillation we found a Q of 1,600,000 for sapphire and 360,000 for Al5056, compared with Q's as high as 5×10^9 achieved for sapphire bars at high frequency[2] and 5×10^7 for Al5056 rods at about 1 kHz[3]. Current data on inelastic behavior of materials[4] suggests that torsional Q's would be independent of frequency, (a result of a frequency-dependent effective damping constant $R(\omega) \propto \omega^{-1}$), while comparing our data for aluminum with reference 3 indicates that Q(1000 Hz)/Q(.01 Hz) $\approx$ 150. Possible explanations for this include: i) effective damping constants in materials may increase more rapidly with lowered frequency than currently believed, ii) losses at fiber clamp points may dominate our measured Q's, and/or iii) surface effects may dominate loss mechanisms for our thin fibers (it has been found[2] for example that polishing the surface of a sapphire bar greatly improves its Q). It is also possible that the operating temperature of the torsion pendulums we tested was significantly higher than that of the 4.2K helium bath which surrounded the pendulums' vacuum chamber, as a result of heating by the light beam of the optical readout system. We plan further investigation of these questions.

3. Dynamic vs static torsion balance operation modes: Noise sensitivity.

A torsion balance may be used in two ways to probe force fields. When the orientation of the instrument relative to the field sources is changed, one may determine either: I. The resulting static deflection of the balance, either in a fixed or continuously rotating frame, or: II. The resulting change in torsional oscillation frequency of the balance, acting as a torsional pendulum. The first method is that used by Cavendish and by Eötvös, and in most recent applications of the torsion balance to tests of the equivalence principle and searches for new long range forces. The second method has been used in several measurements of G, and by one group[5] in searches for new forces. Each method has distinct advantages; here we consider the relative sensitivity of the two methods to seismic and thermal noise when used to measure a differential acceleration Δa of test masses in a force field.

Let $N^2(\omega)$ be the spectral density of a noise torque acting on the balance or pendulum, and define p_c to be the "composition dipole moment" of the balance (dipole moment of one of the two test mass materials relative to the torsion axis). For a measurement made over a time period t the uncertainty in measured Δa introduced by the noise in the two methods is:

$$\delta a(static) = \frac{\sqrt{N^2(\omega_s)}}{p_c\, t^{1/2}} \qquad \delta a(dynamic) \approx \frac{\sqrt{N^2(\omega_0)}}{p_c\, t^{1/2}}$$

Here ω_s is the frequency with which the source field is varied relative to the balance in the static method, and ω_0 is the natural oscillation frequency of the pendulum in the dynamic method. The first equality is easily demonstrated. The expression for the dynamic case is less obvious, but can be shown to be an excellent approximation if three conditions are met: the oscillation amplitude is near an optimum value of 1.84 radians, the measurement extends over several oscillation cycles, and the noise spectrum $N^2(\omega)$ does not have an extreme ω dependence.

Typically a pendulum oscillation period would be on the order of 100 seconds, while the signal frequency in the static method might be a few hours, so that $\omega_0 \approx 100\omega$. Thus whether the static or dynamic method yields lower noise depends on the ω dependence of the noise torque spectrum. Thermal noise arising from the torsion fiber is expected[4] to produce a torque spectrum $N^2(\text{thermal}) \propto \omega^{-1}$, while rotational seismic noise is likely to produce an effective torque spectrum on a torsion balance on the order of $N^2(\omega) \propto \omega^2$. Thus if the dominant noise torque is *thermal*, the dynamic method is likely to have a signal sensitivity *better* by an order of magnitude compared to the static method, while if rotational *seismic* noise dominates, the dynamic method could be *worse* by two orders of magnitude. It is hard to anticipate the level of very low frequency rotational seismic noise to be expected at a quiet field site, so the choice between dynamic and static methods is difficult to make at this point. The limited information available to us on such noise[6] indicates that the dynamic method is favored. Fortunately, an instrument designed for one mode can be readily adapted for operation in the other mode.

4. Effects of fiber non-linearity and anelasticity on measurements of G

Fiber nonlinearity and/or anelasticity are not a significant source of systematic error in null experiments using a torsion pendulum to test the equivalence principle, but are potential error sources in measurements of G such as we plan at UC Irvine[7] using the dynamic method with large amplitude oscillations. Parameterizing nonlinearity in a fiber as a restoring torque related to angular displacement by $N(\theta) = k_1\theta + k_2\theta^2 + k_3\theta^3$, we have determined the ratios k_2/k_1 and k_3/k_1 for a 50 μm diameter 25 cm long aluminum 5056 fiber at 4.2K by analyzing the oscillation frequency $\omega(\theta_0)$ as a function of oscillation amplitude θ_0, and analyzing the harmonic content of the oscillation time dependence $\theta(t)$ for large amplitude oscillation. We find the nonlinearity to be remarkably small: $k_2/k_1 = (-3.1\pm0.4)\times10^{-7}$ and $k_3/k_1 = (-2.7\pm0.7)\times10^{-7}$. If no correction were made for the effects of such nonlinearity in a G measurement, the error introduced in G can be shown to be less than 1 ppm for oscillation amplitudes less than about 4.5 radians, increasing to 4 ppm for an oscillation amplitude of 9 radians. For a 25 μm fiber such as we plan to use in our actual G measurement, nonlinearity effects should be even smaller.

Real springs are found to behave as if they were a set of individual damped springs in parallel, each with its own characteristic relaxation time (the Maxwell model of anelasticity)[8]. Kuroda[9] has noted that such fiber anelasticity can be a significant source of systematic error in measurements of G using the dynamic method. Kuroda evaluated this error for the case discussed by Quinn et al[10] of an anelastic material characterized by a Maxwell model with a continuum spread of relaxation times and a particular choice of relaxation strength parameter. Kuroda also showed that if the imaginary part of the complex torsion constant $k(\omega) = F/x$ is independent of ω, as appears to be approximately the case for real fiber materials, then the fractional bias in G will be equal to $1/\pi Q$.

The continuum Maxwell model is generally accepted as a good model for anelastic material behavior, but a full determination of the distribution of relaxation strengths for a given material would be impossible. Kuroda's important results raise a concern that for some possible relaxation distribution the anelastic fiber behavior might introduce an undetected G bias significantly bigger than $1/\pi Q$. Fortunately, we have found that Kuroda's analysis can be extended to show that the G bias must be bounded between zero and $1/(2Q)$ for *any* distribution of relaxation strengths in the Maxwell model. (This upper bound will be correct to order $1/Q^2$). Thus for a pendulum with $Q \approx 360{,}000$, if a correction $\delta G/G = 1/\pi Q$ is made, any residual error associated with anelastic behavior should be less than 1 ppm.

We conclude from these analyses that neither fiber nonlinearities nor anelastic behavior should preclude a measurement of G with accuracy on the order of 1 ppm, using the dynamic method.

5. Pendulum design for a cryogenic equivalence principle (EP) test

The experiment we envision would compare the effective accelerations of two test mass materials, initially probably magnesium and beryllium, in the fields of the earth and sun. The test masses would be two spheres of each material, resting in holes in a beryllium holder (figure 2). Diamond machined mirror faces on the lower end of this holder serve for optical

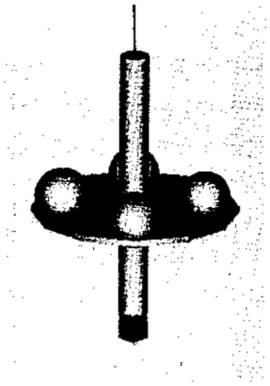

Fig. 2: Pendulum design for a cryogenic equivalence principle test.

position readout. Mass multipole moments of the pendulum are designed to vanish through $\ell=3$. The pendulum would carry 7 gram masses at a radius of 2.4 cm giving a composition dipole moment p_c of 24 g·cm. It would use a 50 μm aluminum 5056 fiber with torsion constant 0.7 dyn·cm/rad, giving a torsional period of about 100 seconds, and would operate in the dynamic mode. As discussed in reference 1, thermal, seismic, and readout noise considerations should allow such an instrument to measure differential accelerations at a level of about 10^{-15} cm/s^2. The key remaining question is how well systematic error from Newtonian gravitational couplings can be controlled. The chief potential source of such error is the coupling of small (nominally zero) m=1 mass multipole moments of the pendulum to gravity field gradients. In principle these moments may be made arbitrarily small by an iterative process in which the pendulum is trimmed by removing small amounts of mass after experimentally determining its response to deliberately augmented gradients. In practice this procedure is limited by the reproducibility of the pendulum's mass distribution at low temperature after a cycle of warming, removal for modification, reinstallation, vacuum bakeout, and recooling. The required reproducability in our design should result from the use of a monolithic mass holder machined as a single piece from beryllium, and spherical test masses which on cooling contract either equally or more than beryllium and thus drop slightly in their holes with reproducibility limited only by deviation from sphericity. Balls of this approximate size can be machined to be spherical within about 1 μin (25 nm), and deviations from spherical shape can be mapped [11] to about 0.1 μin (2.5 nm). Assuming the balls' center of mass lie within 1 μm of their geometrical center, and that the balls are replaced within a milliradian of their original orientation, their center of mass should assume their original position within about 2 nm after a warm/trim/cool cycle. The main effect of ball position shift will be through the coupling of a resulting Q_{xz} type quadrupole moment to an ambient gravity field gradient Φ_{yz}, simulating a differential acceleration of the test masses: $\Delta a = Q_{xz}\Phi_{yz}/p_c$ where p_c is the pendulum's composition dipole moment. For a test mass m a distance ℓ from the axis, displaced vertically by a distance ϵ, Q_{xz} will be $m\ell\epsilon$ while p_c will be $m\ell$ so $\Delta a \approx \epsilon\Phi_{yz}$. In our lab we find we can

readily reduce Φ_{yz} to about 5×10^{-10} s^{-2}; at a field site during dry season it should be possible to maintain Φ_{yz} at a level of 10^{-10} s^{-2}. Assuming $\epsilon \leq 2$ nm leads to an error $\Delta a \approx 4\times10^{-17}$ cm/s^2. Thus we are optimistic that Newtonian gravitational couplings can be controlled at a level that would allow sensitivity at a level $\Delta a = 10^{-15}$ cm/s^2 or better.

6. Comparison with STEP goals

Figure 3 shows limits on a long range force coupling to baryon number that might be obtained with a laboratory instrument with Δa sensitivity of 10^{-15} cm/s^2, compared to present limits and to limits which are the target of space experiments such as STEP. Space EP tests have the advantage of a ~980 cm/s^2 acceleration source, compared to a torsion pendulum's 1.6 cm/s^2 effective field. However the earth-based test we plan has a number of compensating advantages. It can be done at a cost perhaps 1% of that of a space experiment, although probably not more quickly. Checks for systematic errors due to thermal, electromagnetic, and gravitational interactions, by deliberately augmenting the error source, can be made in an operating environment at the design sensitivity level; undesired mass multipole moments may be thus identified and corrected. Charge buildup on test masses from cosmic rays is avoided by use of a conducting torsion fiber, and by operation deep underground if desired. Most importantly, tests for anomalies in the gravitational interaction may be made with high sensitivity in a distance range 10^{-1} to 10^5 m, for which a space experiment is poorly suited.

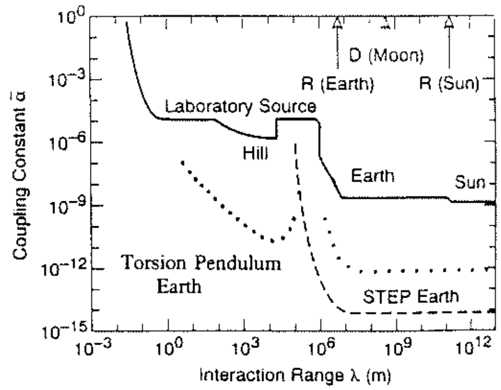

Fig. 3: Limits on a new force coupling to baryon number that might be placed using a torsion pendulum with sensitivity $\Delta a = 10^{-15}$ cm/s^2 (dotted line), compared with existing limits and limits projected for the space-based experiment STEP. Adapted from references 12 and 13.

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Perspectives Pour Expériences de Physique de Gravitation Avec Balances de Torsion Cryogenique

Résumé

Nous discutons les perspectives pour épreuves perfectionnés du principe équivalence faible, recherches des anomalies de gravité Newtonienne (ou forces nouvelles), et mesurages de la constante de gravitation G, utilisant des balances de torsion fonctionnantes à températures cryogeniques. Fonctionnement à températures basses promet les avantages de haut Q mécanique avec bas bruit thermal, haute stabilité mécanique, et haute stabilité à température. Nous discutons problèmes possibles en mesurant G à cause de non-linearité de la fibre et/ou inélasticité, et le problème à minimiser les accouplements de gravitation Newtonienne à une balance utilisée pour une épreuve du principe d'équivalences. Sensitivité à une accélération différentielle des masses d'épreuve au niveau 10^{-15} cm/s^2 ou moins semble un but raisonnable.