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Prospects for terrestrial equivalence principle tests with a cryogenic torsion pendulum

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Abstract

A torsion pendulum may be used to measure effective differential accelerations of test masses in the field of sources on distance scales below those accessible in a space experiment such as STEP. Operation of a torsion pendulum at low temperature (2 K) offers many benefits, notably: low thermal noise, high fibre stability, highly effective superconducting magnetic shielding and excellent temperature control. With such an instrument it should be possible to detect differential accelerations as small as 10^{-14} cm s^{-2}, or fractional differential accelerations in the field of the Earth as small as $\eta = 10^{-14}$. This paper discusses the sources of noise and systematic error that limit a cryogenic torsion pendulum in such measurements.

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1. Introduction

In recent years the focus of equivalence principle (EP) tests has broadened into tests for deviation from composition-independent Newtonian gravity on all distance scales. This is motivated by the fact that nearly all modern theories which show promise for unifying gravity with the other forces of nature predict a violation of the equivalence principle at some level, through interactions mediated by new scalar or vector fields. The range of a possible anomalous interaction varies widely in the various motivating theories, requiring a set of experiments of varying design to probe as large a region in parameter space as possible. For interaction ranges greater than 100 km, by far the most sensitive test of the equivalence principle will be made by STEP (satellite test of the equivalence principle).

For shorter interaction ranges, the torsion pendulum remains the instrument of choice. Since the work of Eötvös [1], the torsion pendulum has been used with progressively increasing sensitivity to test for differential acceleration of materials with differing composition in the gravitational field of source masses at various ranges. The greatest sensitivity has been achieved by the University of Washington 'Eot-Wash' group, whose extensive programme has included tests for differential test mass accelerations in the field of the Sun [2], of the Earth [2], and of laboratory masses [3], with 1σ uncertainties 5.6×10^{-13}, 2.5×10^{-12} and 2.8×10^{-13} cm s^{-2}, respectively.

The torsion pendulum is extraordinary in its ability to measure extremely weak slowly varying forces, but it is very near the limits of its abilities when operated at room temperature. The logical next step is to operate torsion pendulums at cryogenic temperatures. This affords a number of important advantages, principally:

- *Low thermal noise.* The thermal noise torque driving a pendulum is proportional to $\sqrt{k_B T/Q}$, and is thus greatly reduced at 2 K with the high Q achievable at low temperature.
- *High Q.* This is of importance not only for reducing thermal noise, but also for minimizing the effects of fibre anelastic behaviour when a pendulum is disturbed.
- *Frequency stability.* When a pendulum is used in the frequency mode discussed in the following section, variation in its torsional oscillation frequency arising from temperature variation is a serious potential source of both systematic error and random noise. This problem is greatly reduced at low temperature both because the temperature dependence of the fibre's shear modulus and of the pendulum's dimensions becomes very small, and because excellent temperature control to a few μK or better may be maintained.
- *Highly effective superconducting magnetic shielding.*
- *High vacuum.* A very high vacuum is readily maintained in a cryogenic environment with a charcoal cryopump.

A disadvantage of cryogenic operation, however, is the differential contraction of pendulum components on cooling, which makes difficult the design of a mass distribution which maintains the desired symmetry properties when cooled.

At UC Irvine we have been developing cryogenic torsion pendulum technology for a number of years [4–6]. Concurrently, we have studied the theoretical limitations on torsion pendulum performance imposed by thermal, seismic, and readout noise sources [7] and by Newtonian gravitational couplings and the effects of nonlinear and anelastic behaviour of torsion fibres [8]. In this paper we use results of those analyses to estimate the potential performance of an EP test which we plan to conduct in collaboration with Paul Boynton's group at the University of Washington.

2. Pendulum operation modes

Consider a symmetric pendulum carrying symmetrically placed test masses of two types, A and B, in the presence of a field such that masses A and B would, if free, experience horizontal acceleration components $\vec{a} + \Delta\vec{a}$ and $\vec{a}$, respectively. Define the 'composition dipole moment' of the pendulum: $\vec{p}_c = \int \rho_A \, \vec{r}_A \, \mathrm{d}^3\vec{r}$, where the integral is over the distribution of mass type A. Such a pendulum would experience a torque $N(\theta) = N_0 \sin(\theta)$, where θ is an angular displacement relative to the field source and $N_0 = p_c \Delta a$. The ratio of the torque amplitude N_0 to the pendulum's torsion constant k gives a dimensionless measure of the signal: $\epsilon = N_0/k$.

There are three distinct modes in which the pendulum may be used to measure ϵ, as discussed by Paul Boynton [9] whose group introduced two of them. The modes differ dramatically in their relative sensitivity to two major sources of systematic and random error: temperature variation and tilt.

Deflection method. The signal is the displacement $\Delta\theta$ of the pendulum from its equilibrium position, related to ϵ by $\Delta\theta = \epsilon$. The pendulum remains nearly stationary either relative to a fixed instrument, or to a continuously rotating instrument as in the method pioneered by the

University of Washington 'Eot-Wash' group [2]. A big advantage of the continuous rotation variant of the deflection method is its ability to vary the orientation of the pendulum relative to a field source without stressing the torsion fibre. Both variants of the deflection method are quite sensitive to instrument tilt and to temperature variation, putting severe demands on the control of these environmental factors.

Frequency method. Here the signal is a shift in the torsional oscillation frequency of the pendulum, given for small ϵ by $\frac{\Delta\omega}{\omega} = \frac{J_1(A)}{A}\epsilon$, where A is the amplitude of the pendulum's oscillation in radians and J_1 is a Bessel function. Optimal signal-to-noise ratio is achieved for an amplitude A = 1.841. Use of this method in the search for anomalous forces was pioneered by Paul Boynton's group at the University of Washington. With this method, the systematic error arising from instrument tilt is very small compared with the deflection method—a very significant advantage, since instrument tilt associated with varying floor loading or heating is one of the most troublesome error sources in gravity experiments using pendulums. Boynton finds a tilt influence on frequency $\delta \ln(\omega)/\delta\theta_{tilt} \approx 5 \times 10^{-6}$ rad^{-1}. The equivalent sensitivity in a deflection method would be $\delta\theta/\delta\theta_{tilt} \approx 1.5 \times 10^{-5}$ rad rad^{-1}, several orders of magnitude smaller than the sensitivity typically encountered with the deflection method. A serious problem with the frequency method, however, is its sensitivity to temperature variation, which severely limits its usefulness when operating at room temperature.

Harmonic method. Here the signal is a second harmonic distortion of the pendulum's oscillation: $\theta(t) = A\cos(\omega t) + \frac{2}{3}J_2(A)\epsilon\cos(2\omega t) + \cdots$, in response to a torque $N(\theta) = N_0\cos(\theta)$, where again $\epsilon \equiv N_0/k$. This method, developed by Michael Moore in Boynton's group, is extraordinarily insensitive to temperature variations and is even more insensitive to tilt than is the frequency method. A drawback of this method, however, is that it yields a signal-to-noise ratio roughly an order of magnitude less favourable than that of the frequency method.

Choice of method. The temperature sensitivity problem of the frequency method can be overcome if the pendulum is operated at low temperature, making this the method of choice. In the following we assume use of the frequency method.

3. Instrument design

We consider here a specific planned pendulum design, with test masses of magnesium and beryllium. Two spheres of each material rest in holes in a monolithic beryllium holder (figure 1). Diamond machined mirror faces on the upper and/or lower ends of the pendulum serve for optical position readout. Mass multipole moments of the pendulum are designed to vanish through $\ell = 3$. The pendulum would carry 3 g masses at a radius of 2 cm giving a composition dipole moment p_c (dipole moment of one test mass material only) of 8.4 g cm. It would use a 20 μm CuBe fibre with torsion constant 0.03 dyn cm rad^{-1}, giving a torsional period of 290 s with a Q of about 85 000. To achieve geometrical symmetry, the magnesium balls would be of an alloy adjusted to be slightly denser than beryllium. The magnesium and beryllium balls would be machined to be as close to identical in diameter as possible, and then their masses would be equalized by drilling six small symmetrically placed holes in the magnesium balls.

Newtonian gravitational couplings. Among the most worrisome sources of systematic error in an EP experiment is the coupling of small (nominally zero) $m = 1$ mass multipole moments

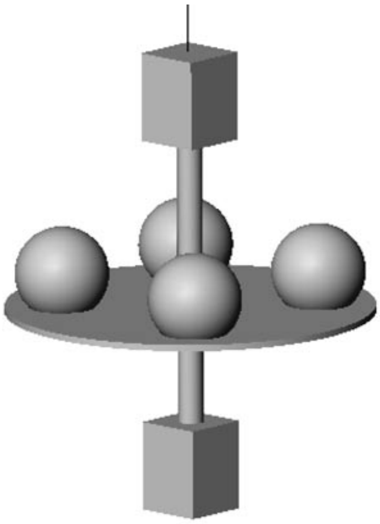

Figure 1. An EP test pendulum design with beryllium and magnesium test mass balls on a beryllium carrier.

of the pendulum to gravity field gradients. In principle, these moments may be made arbitrarily small by an iterative process in which the pendulum is trimmed by removing small amounts of mass after experimentally determining its response to deliberately augmented field gradients. In practice this procedure is limited by the reproducibility of the pendulum's mass distribution at low temperature after a cycle of warming, adjustment and recooling. Reproducibility in our design should result from use of a monolithic beryllium mass holder and spherical test masses which on cooling contract either equally or more than beryllium and thus drop slightly in their holes with reproducibility limited only by deviation from sphericity. Assuming (a) that the balls are round to 25 nm, with centres of mass within 1 μm of their geometric centre, (b) the balls are replaced within 1 mrad of their original orientation and (c) ambient field gradients are kept nulled at a level of 10^{-10} s^{-2}, then we estimate that the simulated differential acceleration resulting from mis-placing the balls after initial nulling would be at most about $10^{-16.5}$ cm s^{-2}.

A related issue is gravitational coupling to an $m = 1$, $\ell = 4$ moment that results if the suspension point of this pendulum is not exactly on its symmetry axis. For example, if the suspension point is displaced laterally by 25 μm, then the gravitational field of a 1 kg mass 1 m away from the pendulum can generate an apparent $\delta a = 2 \times 10^{-15}$ cm s^{-2} as the pendulum is rotated in the laboratory.

A cryostat suitable for an EP pendulum is currently operating in a former Nike missile bunker near Hanford at Richland Washington, where it is being used for a measurement of the gravitational constant G. The cryostat is within a 3 m high LHe dewar which may be rotated on precision bearings under computer control to excite torsional oscillations of a pendulum or to rotate the pendulum's equilibrium position relative to source fields.

Temperature control of the pendulum's environment is maintained in four stages: the vacuum can is maintained at 4.2 K by the main helium reservoir. Within the vacuum can a stage is maintained at 2.0 K by a pumped helium pot. A cylindrical shield, with weak thermal coupling to this stage, is held near 2.1 K by a PID-controlled heater. The pendulum hangs from yet another stage held near 2.15 K by another PID-controlled heater. Temperature control is currently maintained here to within about 20 μK.

Four independent autocollimators view the pendulum. One of these serves to time the oscillation period. Two of the other autocollimators can monitor the small seismically induced tilts of the pendulum in two orthogonal directions, allowing correction of tilt-associated error in the signal from the first autocollimator.

4. Noise sources

The error in differential acceleration measurement is related to the error in frequency measurement by

$$\delta a = \frac{kA}{J_1(A)p_c}\frac{\delta\omega}{\omega}$$

where k, A and p_c are, respectively, the pendulum's torsion constant, oscillation amplitude and the composition dipole moment, or $\delta a \approx \frac{\delta\omega}{\omega} \times 0.012$ cm s^{-2} for the pendulum discussed here. In the following we discuss frequency measurement error and the corresponding δa error in units of $\sqrt{\text{day}}$, i.e. the error for a one day integration time.

The thermal (kT) noise will be dominated by dissipative losses in the fibre material, with a spectrum

$$N^2(\omega) = 4k_BTb(\omega) = \frac{4k_BTk}{\omega Q}$$

where Q is the quality factor of the oscillator at the torsional frequency and $b(\omega)$ is the effective damping resistance at frequency ω, which we assume varies inversely with ω as is typically true for internal friction in metals. This generates torsional frequency noise

$$\frac{\delta\omega}{\omega} = \frac{1}{kA}\sqrt{N^2(\omega)}.$$

For the pendulum considered here this contributes

$$\frac{\delta\omega}{\omega} = 4.5 \times 10^{-12}\sqrt{\text{day}}$$

or $\delta a = 5 \times 10^{-14}$ cm s^{-2} $\sqrt{\text{day}}$.

Readout noise is expected to be dominated by photon shot noise in the autocollimator. This shot noise may be reduced by increasing the power of the autocollimator's light source. However, a fraction of the light power incident on the mirrored pendulum is absorbed and must escape by conduction through the fibre or by radiation. The resulting temperature rise of the pendulum must be limited, thus limiting the light power and hence limiting the reduction in shot noise. With these constraints and the parameters of our optical system, we estimate that shot noise would contribute equivalent torsional frequency noise

$$\frac{\delta\omega}{\omega} = 1.5 \times 10^{-12}\sqrt{\text{day}},$$

or

$$\delta a = 2 \times 10^{-14} \text{ cm s}^{-2} \sqrt{\text{day}}.$$

Rotational seismic noise may produce the dominant component of microseismic noise's contribution to measurement noise. Rotational noise is hard to measure, and little information is available about typical levels. The one estimate available to us (data from a site in New Zealand) suggests a $\frac{\delta\omega}{\omega}$ contribution for period 288 s on the order of $\frac{\delta\omega}{\omega}$ (rotational seismic) $\approx 8 \times 10^{-13}\sqrt{\text{day}}$, or $\delta a \approx 10^{-14}$ cm s^{-2} $\sqrt{\text{day}}$.

Fibre temperature variation is a source of systematic error if the temperature variation is correlated to the modulated signal of interest, but the excellent isolation and temperature control in the cryogenic system makes it unlikely that this will be a serious problem. Of greater concern is the limitation on measurement accuracy from uncorrelated temperature fluctuation noise. This may be estimated from our current fibre's measured dependence of oscillation frequency on temperature,

$$\frac{\mathrm{d}\omega/\omega}{\mathrm{d}T} \approx 5 \times 10^{-6}\ \mathrm{K}^{-1},$$

together with a rough estimate of the pendulum environment's temperature fluctuation spectrum in our current system: 10^{-4} K/$\sqrt{\text{Hz}}$ at $\approx 10^{-4}$ Hz. This indicates $\frac{\delta\omega}{\omega}$ (fibre T variation) $\approx 2 \times 10^{-12}\sqrt{\text{day}}$, or $\delta a \approx 2 \times 10^{-14}$ cm s^{-2} $\sqrt{\text{day}}$.

The tilt of a pendulum's environment is likely to be correlated to a modulated signal of interest and is often a serious problem in experiments using a torsion pendulum. Fortunately the frequency method is relatively insensitive to tilt. A room-temperature pendulum currently operated by Boynton's group has a measured tilt sensitivity [10]

$$\frac{\delta\omega}{\omega} = 5 \times 10^{-6}\ \mathrm{rad}^{-1}.$$

Assuming that tilt can be monitored and controlled at a level of 10^{-8} rad, this implies a maximum signal error

$$\frac{\delta\omega}{\omega} = 5 \times 10^{-14},$$

or $\delta a = 5 \times 10^{-16}$ cm s^{-2} (an error that need not decrease with run time).

Possible improvements. If mirrored surfaces on the pendulum were made so that they absorb only 0.001 of the incident light, higher power could be used in the autocollimators, reducing the photon shot noise by a factor of 10. Higher Q may be possible. Duffy [11] has found that the Q at 4 K of CuBe resonators at about 1 kHz is increased dramatically by heat treatment, from about 10^5 to 3×10^6. Our own first try with a heat-treated CuBe fibre yielded negligible Q increase. The surface of Duffy's resonators had been treated with an acid etch; we will try acid etching and/or polishing of our fibre's surface to see if this may raise our Q to the level observed in resonators at high frequency.

5. Projected and present sensitivity

If other systematic error sources are not a limiting factor, the combined error estimates above suggest that with the presently planned pendulum parameters it may be possible in a 100 d run to reach a sensitivity $\delta a < 10^{-14}$ cm s^{-2}, while with higher Q and better mirrors the sensitivity might be pushed to $\delta a \approx 10^{-15}$ cm s^{-2}.

As a reality check, we may consider the demonstrated performance of our present torsion pendulum which operates at 2 K for a measurement of the gravitational constant G. This pendulum is an 11 g square quartz plate suspended with a 20 μm CuBe fibre, oscillating with amplitude 2.57 rad and period 135 s. With this pendulum we can measure an oscillation frequency shift modulated with 2 h period with sensitivity (scaled to the 1.84 rad amplitude for an EP experiment)

$$\frac{\delta\omega}{\omega} \approx 1.2 \times 10^{-10}\sqrt{\text{day}}.$$

For the projected EP pendulum, this would translate to $\delta a \approx 1.4 \times 10^{-12}$ cm s$^{-2}\sqrt{\text{day}}$. This is about 25 times worse than would be expected from the noise sources discussed above, and comparable to the performance of the best room-temperature pendulums [3] but not significantly better. We have not yet identified the source of our excess noise. We hope that it is linked to the deliberately very large quadrupole moment of the G pendulum, through couplings to time-varying ambient gravity gradients and/or couplings between swing and torsional oscillation modes—in which case this excess noise will be largely absent with the highly symmetric pendulum used in an EP test. Other possibilities include noise associated with the flow of helium into the 2 K pot through the capillary tube (for which there are cures [12]), and noise linked to vertical bounce mode oscillations of the pendulum, which may be reduced with appropriate damping [13]. The observed noise does not appear to be particularly correlated to ambient microseismic noise monitored by a seismometer.

We are optimistic that a sensitivity of $\delta a \approx 10^{-14}$ cm s^{-2} may be achieved with a cryogenic torsion pendulum. To illustrate what might be accomplished with an instrument of this sensitivity, figure 2 shows the 2σ limits that could be placed on a new force coupling to baryon number:

$$V_{12} = \frac{-Gm_1m_2}{r}\left[1 + \alpha\left(\frac{B}{\mu}\right)_1\left(\frac{B}{\mu}\right)_2 \mathrm{e}^{-r/\lambda}\right] \tag{1}$$

where $(B/\mu)_i$ is the ratio, for mass i, of the number of its nucleons to its mass in amu, and α and λ are, respectively, the strength and range of the new force. Also shown in figure 2 are existing limits on such a force, and the sensitivity projected for the space experiment STEP.

The projected sensitivity labelled 'lab source' in figure 2 assumes a 2 ton copper source mass which is being built by Boynton's group, initially to be used for a test of the gravitational inverse square law. For an EP test this source mass would be rotated at intervals around the pendulum as it oscillates in a stationary cryogenic housing.

The projected sensitivity labelled 'mountain' assumes as field source a basalt mountain that rises 700 m above our missile bunker site. The projected sensitivity labelled 'earth' assumes an 'Eötvös' style experiment using the Earth as a whole as the field source. To make either of these tests, the pendulum together with its housing must be rotated at periodic intervals in the laboratory. Due to the anelastic properties of the torsion fibre and the glue in which it is mounted, this instrument rotation causes a small temporary shift in pendulum oscillation frequency which dies away on a mixture of time scales. The magnitude of this effect is inversely proportional to the Q of the fibre, and hence is reduced by using a fibre with high Q. Furthermore, if the velocity pattern $\dot{\theta}(t)$ of the instrument's rotation is precisely the same as it is periodically rotated first from 0 to π and then from π to 0, there should be no effect on the frequency *difference* measured in the two instrument orientations. Such precise rotation control should be achievable using a high-precision angular encoder. Nevertheless, fibre anelastic effects may be the limiting factor in using a torsion pendulum in frequency mode to improve equivalence principle test sensitivity over distance ranges from 10^2 m to 10^{11} m.

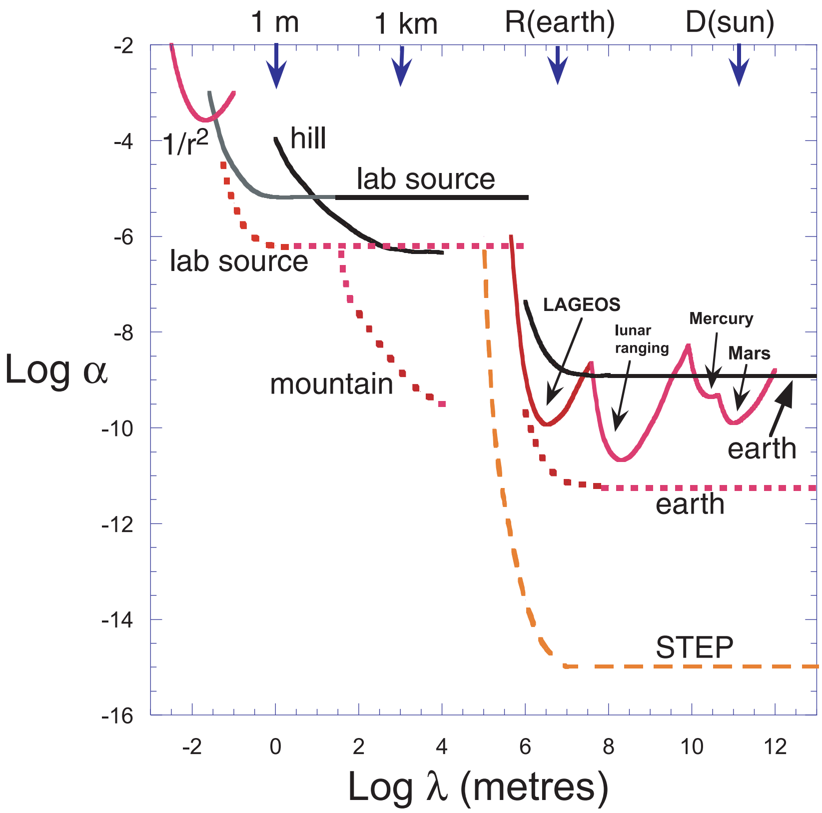

Figure 2. Present and potential 2σ constraints on a new force coupling to baryon number (equation (1)). The full curves labelled 'lab source', 'hill' and 'earth', are existing constraints from the work of the Eot-Wash group [2,3]. The curve labelled $1/r^2$ is a constraint from an inverse square law test [14]. The curves labelled LAGEOS, lunar ranging, Mercury, and Mars are from Nordtvedt's analysis of planetary and satellite orbital data [15]. The broken curve is the projected constraint ability of STEP. The dotted curves are constraints that might be achieved with a torsion pendulum having sensitivity $\delta a = 10^{-14}$ cm s^{-2}.

For ranges above 10^{11} m, the Sun may be used as the field source. In this case an EP test may be made using the Earth's rotation to modulate the signal, avoiding the need to disturb the instrument by rotating it in the laboratory.

Cryogenic operation seems to be the only option for significantly improving the present performance of torsion pendulums for equivalence principle tests. Whether such improvement can be realized remains to be demonstrated, but a sensitivity $\delta a = 10^{-14}$ cm s^{-2} appears to be a reasonable goal.

Acknowledgments

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